MITL - A 2-D Code to Investigate Electron Flow Through Non-Uniform Field
Region of Magnetically Insulated Transmission Lines*

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Abstract

Self-magnetically insulated, high voltage transmission lines are used in inertial confinement fusion particle accelerators[1] to transmit power from the vacuum insulator to the diode. Injection and output convoluted sections pose special problems in establishing the desired electron flow pattern needed to maintain high overall efficiency. A time independent, 2-D numerical code for planar or triplate geometries calculates the motion of a test electron through the tapered input or output convolutes. The 1-D parapotential model is assumed to be appropriate at each position and the magnetic field and potential distribution are calculated in the vicinity of the particle. The electric field is then calculated from Gauss's Law, and the electron motion is calculated relativistically. The results show that the electron canonical momentum in the direction of flow changes as the electron passes through a convoluted geometry. As shown by Mendel,[2] these electrons flow between conductors after the convolute without re-intersecting the cathode. We hypothesize that these electrons lead to the losses observed in long self-magnetically insulated lines. Results of calculations are correlated with results of the Mite power flow experiment.[3]

Introduction

Transition sections into the magnetically insulated transmission lines can excite an apparent instability in the electron flow within the transport section[3,4] and cause severe energy losses. A numerical, time independent 2-D code, MITL, has been written to investigate the effects of these transition sections on the flow pattern within the transport section. The code is used to examine the input transition in the Mite experiment.[3,4]

The results suggest that the input transitions produce electron flow in which the axial canonical momentum P_x is approximately 10^{27} kg-m/s or 10^{-6} of that allowed in the line. The transitions that produce broad canonical momentum distributions $F(P_x)$ are correlated with efficient power transport in the experiments. Those that produce narrow distributions are correlated with lossy transport experimentally.

Description of the Problem and the Approach

The rectangular geometry used in MITL is appropriate to triplate type transition sections and is known in Fig. 1. The effective width w of the lines perpendicular to $\bar{E}$ and $\bar{E} \times \bar{B}$, and the conductor separation d are variables that specify the geometry of either the input or the output transition section. Time independence is assumed in MITL since the transit time through the transition sections are typically a few nanoseconds and are much shorter than the pulse duration. Electromagnetic fields within the magnetically insulated lines are calculated on the assumtion that the 1-D parapotential equations derived by Creedon[5] are appropriate for each position in the 2-D convolute. The justification for using the 1-D parapotential model is two fold: First, the scale length for the geometry variation is long compared to the separation between conductors so the flow is approximately one dimensional at each position. Secondly, the parapotential theory adequately describes the relationship between the total current I_T, the boundary current I_B inside the negative conductor, and the voltage V_0 for a given line with a vacuum wave impedance Z_0. The agreement with 2-D electromagnetic PIC simulations[6] and experiments[3,6] is within the numerical and experimental uncertainties respectively. The electromagnetic fields from the 2-D calculation[6] for a Mite like line at 2.4 MV with I_T = 450 kA and I_B = 243 kA in a coaxial geometry with Z_0 = 8 Ω, have been compared in Refs. 6 and 7, and the agreement justifies the use of parapotential theory in these calculations.
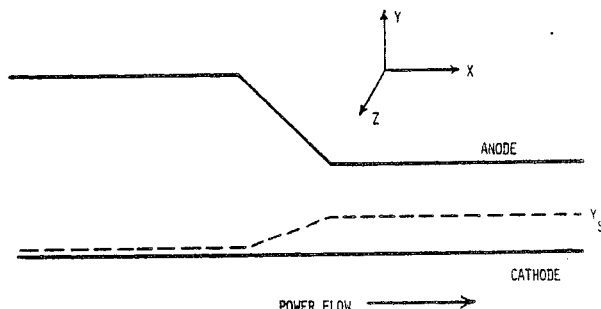

Fig. 1. Transition section geometry in MITL. The
line width is specified in the Z direction

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14. ABSTRACT

Self-magnetically insulated, high voltage transmission lines are used in inertfal confinement fusion particle accelerators to transmit power from the vacuum insulator to the diode. Injection and output convoluted sections pose special problems in establishing the desired electron flow pattern needed to maintain high overall efficiency. A time independent, 2-D numerical code for planar or triplate geometries calculates the motion of a test electron through the tapered input or output convolutes. The 1-D parapotential model is assumed to be appropriate at each position and the magnetic field and potential distribution are calculated in the vicinity of the particle. The electric field is then calculated from Gauss's Law, and the electron motion is calculated relativistically. The results show that the electron canonical momentum in the direction of flow changes as the electron passes thro~gh a convoluted geometry. As shown by Mendel, these electrons flow between the conductors after the convolute without re-intersecting the cathode, We hypothesize that these electrons lead to the losses observed in long self-magnetically insulated lines. Results of calculations are correlated with results of the Mite power flow experiment. 3

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The test electron is then injected into the convolute and its motion followed through the convolute and into the uniform self-magnetically insulated transmission line. Once inside the uniform line, the canonical momentum

$$P_x = \gamma_e m U_x - e A_x \qquad (1)$$

is a constant of its motion, where m is the electron rest mass, -e is the electron charge, U_x is its axial velocity, A_x is the axial component of the magnetic vector potential and

$$\gamma_e \equiv 1/(1 - U^2/c^2)^{1/2} \qquad (2)$$

for an electron of speed U and $c = 3 \times 10^8$ m/s. The sum of its kinetic and potential energy is also a constant of its motion. Since the problem is assumed to be electrostatic, the energy is not changed by the convolute. However, since $dL/dx \neq 0$ in the convolute, then P_x is changed by ΔP_x according to Lagrange's equation as the electron moves through the convolute. Consequently, the problem is reduced to calculating ΔP_x accurately. Generally, the limitations of finite cell size and a finite number of particles in 2-D self-consistent simulations severely limit the accuracy with which ΔP_x can be computed. The numerical noise is avoided, at the expense of self-consistency, by using analytic equations for $\bar{E}$ and $\bar{B}$. Without self-consistency the calculations are not, however, quantitatively exact. The value of these calculations is the insight they provide into the effect of convolutes on the electron flow.

Description of the Program

The program involves the choice of the initial conditions for the electron as it leaves the cathode plasma in the convolute, the calculation of the electric $\bar{E}$ and magnetic $\bar{B}$ fields in the vicinity of the test electron, and the integration of the relativistic equation of motion as the particle progresses through the convolute and the uniform transmission line. Each feature will be discussed and then the results of the Mite calculations will be presented.

The test particles are assumed to originate in a cathode plasma at a y coordinate $y_o = 10^{-6}$ m, where the voltage is ≤ 0.3 eV and the magnetic vector potential is A_o, which is calculated from the parapotential theory.[5] The initial energy of the electron is chosen between 1 and 10 eV to simulate the effect of electron emission from a few eV plasma. The initial particle energy determines the absolute value of the electrons initial velocity U_o. The initial canonical momentum $\bar{P} = P_x \hat{x}$ is assumed to be zero, so initially,

$$U_x = \frac{e A_o}{\gamma_e m}$$

$$U_y = \sqrt{U_o^2 - U_x^2} \qquad (3)$$

The initial value of $X = X_o$ is chosen for each calculation.

The total current flowing through a magnetically insulated line is given by

$$I_T = I_\alpha g \gamma_m \left\{ \ln [\gamma_m + (\gamma_m^2 - 1)^{1/2}] + \frac{\gamma_o - \gamma_m}{(\gamma_m^2 - 1)^{1/2}} \right\} \qquad (4)$$

$$\gamma = \frac{eV}{m_o c^2} + 1 \qquad (5)$$

where γ_m is the value of γ at Y_m the edge of the flow pattern, and γ_o is given by the applied line voltage. The local line geometry determines the value of

$$g = \frac{60}{Z_o} = \frac{w(x)}{2\pi d(x)} \qquad (6)$$

The input parameters are the line profile w(x) and d(x) for each axial position x, the voltage V_o at the anode for V = 0 at the cathode, and the total current I_T through the structure. For a given position (x,y), the value of $\gamma_m(x)$ is calculated from Eq. 4. The voltage V(x,y) is given by

$$V = \frac{mc^2}{e} \left(\cosh \frac{Y}{C_1} - 1 \right) \qquad 0 \leq Y \leq Y_m \qquad (7a)$$

or

$$V = \frac{mc^2}{e} \left(\frac{\gamma_o (Y - Y_m) - \gamma_m (Y_T - d)}{d - Y_m} \right) \qquad Y_m < Y \leq Y_o \qquad (7b)$$

where

$$C_1 = d I_\alpha g \gamma_m / I_o \qquad (8)$$

and

$$Y_m = C_1 \ln [\gamma_m + (\gamma_m^2 - 1)^{1/2}] \qquad (9)$$

where Y_m is the position of the sheath, Y_o is the position of the anode and the cathode is at Y = 0. The values of V at four positions equally spaced about (x,y) are calculated and the electric field is calculated from

$$\bar{E} = -\bar{\nabla} V \qquad (10)$$

The local magnetic field $\bar{B} = B_z \hat{z}$ is given by

$$B_z = -\frac{mc^2}{e} (\gamma - 1) \qquad 0 \leq Y \leq Y_m \qquad (11a)$$

$$B_z = -\frac{\mu_o I_T}{w} \qquad \text{if } Y_m < Y \leq Y_o \qquad (11b)$$

The relativistic equation of motion for the electrons is

$$\frac{d}{dt} (\gamma_e M \bar{U}) = -e (\bar{E} + \bar{U} \times \bar{B}) \qquad (12)$$

and is combined with the local electric and magnetic fields. It is then solved using the integration routine STFODE[8] to find the new particle postion and velocity components, within a given error criteria, after a time increment. The particle is progressively accelerated through the transition and transport sections of line for successive time steps.

Results

The Mite experiment[3,4] used two transition geometries to change the gap spacing from 0.02 to

0.01 m in a triplate vacuum transmission line with an effective width w = 0.50 m. In the first geometry, the transition was made over an axial length of 0.04 m and 40 percent of the power was lost between 0.50 and 1.4 m from the beginning of the uniform line. The second transition was made over an axial length of 0.14 m and the power transport was about 100 percent efficient. These two geometries were simulated with typical Mite parameters of V_o = 2.0 MV, I_T = 0.4 MA.

The effect of having space charge in the vacuum gap is illustrated in Fig. 2. The equipotentials for a 1 cm taper are shown for the Mite parameters of V_o and I_T. The position of the edge of the electron sheath is Y_m and is shown. The effect of the space charge is to produce a positive E_x near the cathode and to distort the distribution of E_y and B_z.
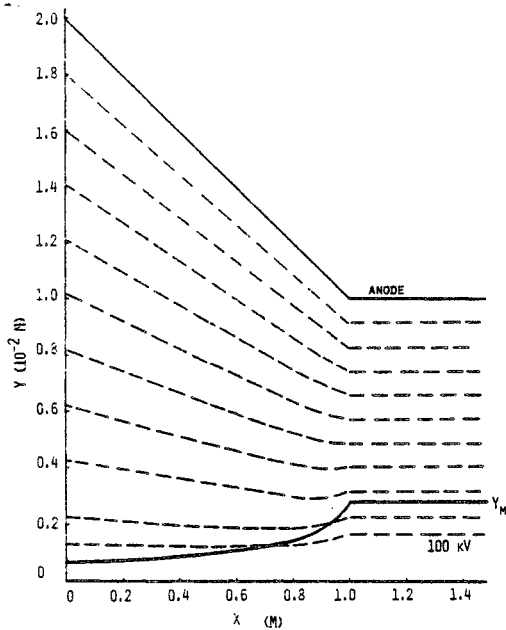

Fig. 2. Effect of space charge in vacuum gap of 1 cm long transition is shown. The dotted lines are equipotential at 200 kV intervals and Y_m is the electron sheath position.

For the severe 0.01 m long transition the distortion is not very large. The maximum value of $E_x/(V_o/d)$ is only 0.005 for the 1 cm taper and is much less for the 0.04 m and 0.14 m tapers. Consequently, the effect of the convolute on the electron motion is small and must be calculated with a very small relative and absolute errors of $\delta = 10^{-6}$ $\overline{X}$.

The current I_s carried by the electron flow increase steadily as the spacing between conductors is reduced in the transition section, as shown in Fig. 3 for the 0.14 m taper. The final canonical momentum that the electron achieves as it is accelerated through the convolute is shown as a function of its initial position X_o in Fig. 4 and 5 for the 0.04 m and 0.14 m convolutes respectively.

The emission current as a function of X_o is also shown. Since the differential electron current $J_s(x) = \Delta I_s/\Delta X$ is supplied from the cathode, the approximate shape of the canonical momentum distribution $F(P_x)$ can be approximated by

$$F(P_x) \simeq \frac{1}{\gamma_m \, wec} \frac{J_s(x(P_x))}{dP_x/dx}$$

which is shown in Fig. 6 for both tapers under the assumption that the initial energy W_o of the electrons is 10 eV at the cathode surface.
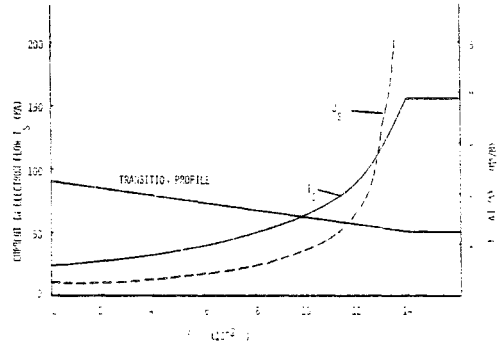

Fig. 3. The electron current I_s and the calculated emission current per unit length for the 0.14 m long transition is shown.
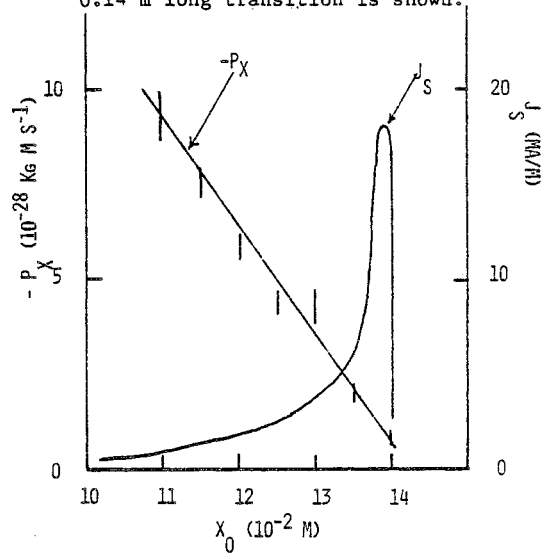

Fig. 4. The final canonical momentum P_x and the electron emission current J_s vs. the initial electron position X_o for the 0.04 m taper.

Discussion and Conclusions

In both cases, the canonical momentum is negative and $F(P_x)$ has a very small width. The spread in P_x is $\approx 10^{-6}$ of that allowed in the uniform line.[7] The small values of P_x obtained with space charge are much less than the values estimated from the vacuum fields alone.[9]
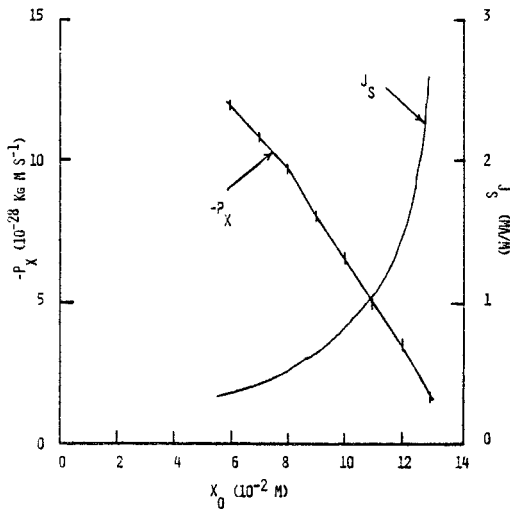
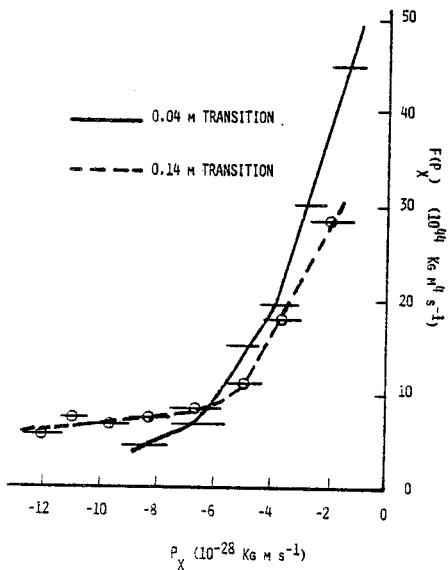Fig. 5. Plot of P_x and J_s vs. X_o for the 0.14 m taper.

Fig. 6. The calculated distributions $F(P_x)$ for the Mite transition sections.

The fact that $P_x < 0$ for the injected electrons means that they will flow for many Larmor radii. Electron with $W_o = 10$ eV travel through the line past the 0.50 m position at which the losses occur and could be susceptible to an instability with a spatial growth length much less than 0.50 m, as hypothesized[4] to explain the Mite results.

Since the electron current is a very strong function of the separation between plates, the bulk of the electrons in the flow have small values of P_x. Even though the gradients in the fields are larger near the end of the transition, the electrons are accelerated through the transition section before they acquire a large negative canonical momentum.

The electrons with more negative values of P_x originate further from the output of the transition section. The effect of using a more gradually tapered transition section is to broaden the canonical momentum distribution. Since the more gradual taper has efficient power propagation, the results indicate that a broader $F(P_x)$ provide more reliable transport in long lines. An injector designed such that J_x approximately equals a constant through a long transition section should be the optimum arrangement. Such a transition section will be designed and tested on the Mite experiment.

In conclusion, the simulations indicate that the difference between the distributions $F(P_x)$ for the lossy and the efficient transitions is small but significant. The results suggest a way to improve the transition and define experiment and theory development required to explore the implications further. The results indicate that an experiment[7] to measure $F(P_x)$ for the Mite transition sections should be capable of resolving the distributions in Fig. 6. Finally, the stability of electron flow with $F(P_x)$ similiar to those presented in Fig. 6 should be examined under the conditions of the large E_y and B_z present in self magnetically insulated transmission lines.

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